Comparison of Mixture and Relative Models in DIF Detections via Two-Level Random-Effect Logistic Regression: A Bayesian Approach

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Outline

- Problems
- Purpose of the study
- Methods
- Simulation - Conditions
- Simulation - Preliminary Results
- Further investigations
- References

Problems

- Scale indeterminancy of DIF parameters:
  - DIF parameters can be scaled only by providing a scaling reference point.
  - DIF parameters are "ipsative" measures.
    - Scale of DIF parameters are all relative to each other.
  - The scale of the mean ability difference is also related to the scale of DIF parameters.

Problems

Example:

- Gender DIF on TIMSS Data from Wu et al. (1997)
  - 6 science items – all dichotomously scored.
  - 6,800 students in the first 2 years of secondary schooling in Australia.

	Cheong (2006)	Kamata (1999)	Mantel Haenszel
Ability difference between girls and boys	−.11	.32	Not available
Item 1	−.12	−.55**	-.11
Item 2	−.06	−.49**	-.07
Item 3	.20**	−.23**	.29**
Item 4	−.58**	− 1.01**	-.70**
Item 5	.12**	−.31**	.14**
Item 6	.44**	0	.49**

Problems, continued

- Three possible solutions

1. Select a valid "anchor set" to establish a valid matching criterion.
    - If the matching criterion is truly bias free, DIF estimates are also bias free.

Problems, continued

2. Parameterize DIF parameters, such that the mean of DIF magnitudes is the reference point.
   - Magnitudes of DIF parameters are deviation scores from the mean DIF in the test.
   - DIF parameter estimates are "unbiased", only when (mean DIF) = 0.
   - This approach is equivalent to Cheong (2006), and referred to as "relative DIF" in this study.

Problems, continued

3. Constrain one or more item's DIF to a fixed value. – Use of reference item(s).
   - Typically, DIF of reference items are fixed to 0.
   - Magnitudes of DIF parameters for other items are deviation from the DIF magnitude of the reference item.
   - DIF parameter estimates are "unbiased", only when (DIF of reference item) = 0.
   - This approach is equivalent to Kamata (1999).

Mixture-DIF IRT model

- Originally proposed by Chaimongkol (2005).
- Treat DIF parameters as a random variable with a mixture distribution of
  - Normal distribution
  - Bernoulli distribution
- It is a "latent class" modeling with 2 latent classes of items.
  - Contrary to a typical latent class modeling, latent classes are for items rather than for examinees.

Mixture-DIF IRT model

- Conceptually, items are divided into two latent classes depending on the magnitude of DIF; items with zero DIF and items with non-zero DIF.
- Then, DIF parameters are estimated based on the mixture distribution of the DIF parameter.
- No explicit constraint is required for DIF parameter estimation.
- Therefore, this method may be useful for estimating "unbiased" DIF magnitudes without specifying a "correct" scaling reference point explicitly.

Purpose of the study

- Chaimongkol (2005) only demonstrated the recovery of mixture-DIF parameters in a very simple context.
- This study conducts extensive simulation study to explore the utility of the model, especially when the relative-DIF approach does not work well.

Methods- 2-Level DIF Detection Model

$$x_{ij} \sim Bernoulli(p_{ij})$$

$$\log\left(\frac{p_{ij}}{1-p_{ij}}\right) = u_{2i} + \alpha_0 G_i - \beta_j - \gamma_j G_i$$

- $i = 1, \ldots, N$ students and $j = 1, \ldots, J$ items
- $u_{2i} \sim N(\mu, \sigma^2_{G_i})$: the ability of student i
- α_0 : the mean ability difference between groups
- β_j : item difficulty when student i is in the reference group
- G_i : 0 for reference group, 1 for focal group
- $\gamma_j \sim N(0, \sigma^2)$: DIF against focal group when it is negative

Methods- 2-Level DIF Detection Model - Model Identification

$$x_{ij} \sim Bernoulli(p_{ij})$$

$$\log(\frac{p_{ij}}{1-p_{ij}}) = u_{2i} + \alpha_0 G_i - \beta_j - \gamma_j G_i$$

Option 1(Reference item)

Fix mean of ability (μ_{u2i}) and DIF for item j ($\mu_{\gamma j}$) to zero

Option 2 (Relative)

Constrain overall mean DIF value (across all items) to zero $(\sum \gamma_k = 0)$

So, the DIF magnitude for any item in the test is detected relative to mean overall DIF for all of the items in the test.

Methods- 2-Level Relative DIF Detection Model*

$$x_{ij} \sim Bernoulli(p_{ij})$$

$$\log(\frac{p_{ij}}{1-p_{ij}}) = u_{2i}^{adj} + \alpha_0^{adj} G_i - \beta_j^{adj} - \gamma_j^{adj} G_i$$

$$u_{2i}^{adj} = u_{2i} - \bar{\beta}$$
$$\alpha_0^{adj} = \alpha_0 - \bar{\gamma}$$
$$\beta_j^{adj} = \beta_j - \bar{\beta}$$
$$\gamma_j^{adj} = \gamma_j - \bar{\gamma}.$$

→ Here option 2 is applied

$$\mu^{adj} = \mu - \bar{\beta}$$

→ Mean (μ) of the ability parameter is also adjusted by subtracting mean of the difficulties($\bar{\beta}$)from it

*Bafumi, Gelman, Park, and Kaplan (2004)

Methods- 2-Level Mixture-Item DIF Detection Model*

$$x_{ij} \sim Bernoulli(p_{ij})$$

$$\log\left(\frac{p_{ij}}{1-p_{ij}}\right) = u_{2j}^{adj} + \alpha_0 G_i - \beta_j^{adj} - \gamma_j G_i$$

- $i = 1, \ldots, N$ students and $j = 1, \ldots, J$ items
- $u_{2i} \sim N(\mu, \sigma_{G_i}^2)$: the ability of student i
- γ_j has a mixture distribution now:
  - Bernoulli part: probability (p) that each item displays a non-zero DIF magnitude
  - Normal part: the sampling distribution of the magnitude of DIF on a continuous scale

$$\gamma_j = \begin{cases} N(0, \sigma^2) & \text{with probability } p; \\ 0 & \text{with probability } (1-p). \end{cases}$$

→ Items with non-zero DIF

→ Items with zero DIF

*Chaimongkol (2005)

Methods- 2-Level Mixture-Item DIF Detection Model*

$$x_{ij} \sim Bernoulli(p_{ij})$$

$$\log\left(\frac{p_{ij}}{1-p_{ij}}\right) = u_{2j}^{adj} + \alpha_0 G_i - \beta_j^{adj} - \gamma_j G_i$$

- The distribution of DIF magnitudes for items with zero DIF is a very tight distribution around 0,
- The distribution for non-zero DIF is a reasonably wide distribution where its mean is driven by the data.
- Important Point:
  - Mixture-Item DIF detection model does not require any explicit scale constraint to estimate DIF parameters, due to the imposed mixture distribution of DIF parameter.

*Saengla Chaimongkol (2005)

Simulation Study- Conditions

- Test Length : 10
- Number of DIF items : 4
- DIF magnitude : ± 0.3, 0.6 log-odds-ratio,
  - Non-zero mean DIF
  - Zero Mean DIF
- Sample size : 2000
- Initial value of p for Bernoulli : 0.1, 0.5, or 0.9
- Replication : 100 data sets
- Parameter estimation : MCMC with WinBUGS

Simulation- Results 1

Parameter Estimates: *n*=2000, Non-zero Mean DIF(0.3, 0.6), 4 items

Parameter	True value	Mixutre DIF						Relative DIF	
		prior p=0.1		prior p=0.5		prior p=0.9			
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
$\alpha 0$	0.3	0.25	(0.07)	0.25	(0.07)	0.25	(0.06)	0.10	(0.06)
β_1^{adj}	-1	-0.96	(0.08)	-0.96	(0.09)	-0.97	(0.09)	-0.85	(0.13)
β_2^{adj}	-1	-0.97	(0.08)	-0.97	(0.09)	-0.96	(0.08)	-1.01	(0.13)
β_3^{adj}	-1	-0.99	(0.07)	-0.99	(0.07)	-0.99	(0.08)	-1.09	(0.06)
β_4^{adj}	0	-0.01	(0.07)	0.01	(0.07)	0.02	(0.07)	-0.06	(0.04)
β_5^{adj}	0	-0.02	(0.05)	-0.01	(0.06)	-0.01	(0.06)	0.05	(0.16)
β_6^{adj}	0	-0.01	(0.06)	-0.02	(0.05)	-0.02	(0.06)	-0.11	(0.01)
β_7^{adj}	0	-0.01	(0.06)	-0.01	(0.05)	-0.01	(0.06)	0.09	(0.02)
β_8^{adj}	1	0.99	(0.05)	0.99	(0.05)	0.98	(0.06)	0.98	(0.08)
β_9^{adj}	1	0.98	(0.06)	0.98	(0.05)	0.98	(0.06)	1.07	(0.13)
β_{10}^{adj}	1	0.98	(0.05)	0.99	(0.06)	0.99	(0.06)	0.94	(0.14)

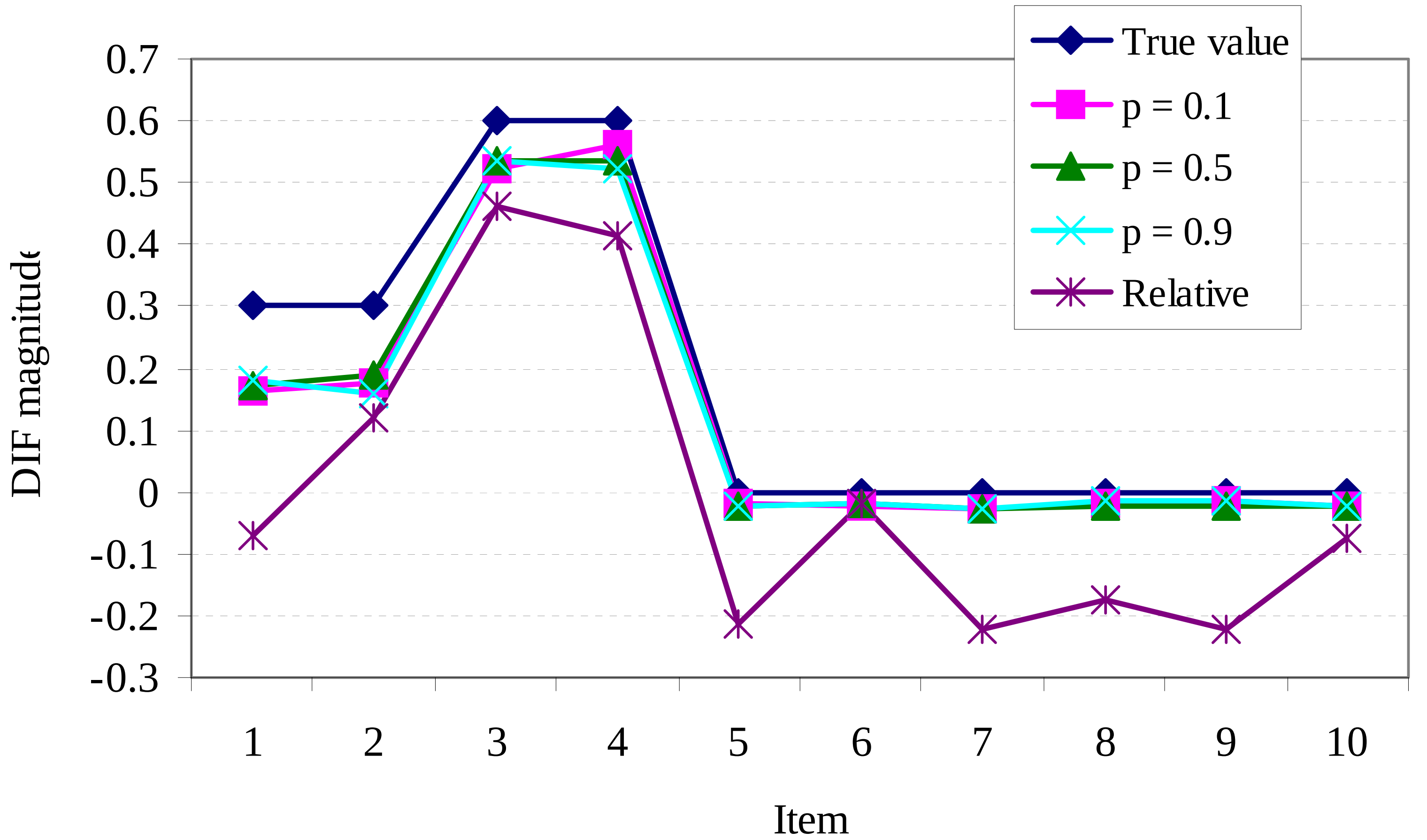
Simulation- Results 1-Cont.

Parameter Estimates: n=2000, Non-zero Mean DIF(0.3, 0.6), 4 items

Parameter	True value	Mixutre DIF						Relative DIF	
		prior p=0.1		prior p=0.5		prior p=0.9			
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
γ_1	**0.3**	**0.16**	**(0.14)**	**0.17**	**(0.15)**	**0.18**	**(0.16)**	**-0.07**	**(0.16)**
γ_2	**0.3**	**0.18**	**(0.14)**	**0.19**	**(0.15)**	**0.16**	**(0.16)**	**0.12**	**(0.14)**
γ_3	**0.6**	**0.52**	**(0.13)**	**0.53**	**(0.15)**	**0.54**	**(0.13)**	**0.46**	**(0.14)**
γ_4	**0.6**	**0.56**	**(0.13)**	**0.54**	**(0.12)**	**0.52**	**(0.12)**	**0.42**	**(0.04)**
γ_5	0	-0.02	(0.04)	-0.02	(0.06)	-0.02	(0.05)	-0.21	(0.02)
γ_6	0	-0.02	(0.06)	-0.02	(0.04)	-0.02	(0.06)	-0.02	(0.06)
γ_7	0	-0.03	(0.08)	-0.03	(0.06)	-0.03	(0.08)	-0.22	(0.02)
γ_8	0	-0.02	(0.05)	-0.02	(0.05)	-0.02	(0.06)	-0.18	(0.11)
γ_9	0	-0.01	(0.05)	-0.02	(0.06)	-0.01	(0.04)	-0.22	(0.11)
γ_{10}	0	-0.02	(0.07)	-0.02	(0.06)	-0.02	(0.05)	-0.07	(0.32)
p	-	0.43	(0.08)	0.42	(0.08)	0.42	(0.09)	-	-
σ^2_1	1	1.00	(0.04)	1.00	(0.04)	1.00	(0.04)	1.01	(0.09)
σ^2_2	1	1.01	(0.04)	1.01	(0.04)	1.01	(0.04)	1.01	(0.09)

Simulation- Results 1 Cont.

Parameter Estimates: n=2000, Non-zero Mean DIF(0.3,0.6), 4 items



Simulation- Results 2.

Parameter Estimates: *n*=2000, Non-zero Mean DIF(0.6), 4 items

Parameter	True value	Mixutre DIF						Relative DIF	
		prior p=0.1		prior p=0.5		prior p=0.9			
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
$\alpha 0$	0.3	0.26	(0.11)	0.25	(0.13)	0.26	(0.11)	0.06	(0.06)
β_1^{adj}	-1	-0.99	(0.07)	-0.99	(0.08)	-0.99	(0.07)	-1.00	(0.07)
β_2^{adj}	-1	-1.00	(0.07)	-0.99	(0.08)	-0.99	(0.07)	-1.00	(0.07)
β_3^{adj}	-1	-0.98	(0.07)	-1.00	(0.08)	-0.98	(0.07)	-1.01	(0.07)
β_4^{adj}	0	0.00	(0.06)	0.00	(0.07)	0.02	(0.07)	0.00	(0.06)
β_5^{adj}	0	-0.01	(0.05)	0.00	(0.06)	-0.01	(0.06)	0.01	(0.06)
β_6^{adj}	0	0.00	(0.06)	-0.01	(0.05)	-0.01	(0.06)	0.00	(0.06)
β_7^{adj}	0	0.00	(0.06)	-0.01	(0.06)	-0.01	(0.06)	-0.01	(0.06)
β_8^{adj}	1	1.00	(0.06)	0.99	(0.06)	0.99	(0.07)	1.02	(0.07)
β_9^{adj}	1	0.99	(0.06)	0.99	(0.06)	0.99	(0.06)	1.01	(0.07)
β_{10}^{adj}	1	1.00	(0.06)	1.01	(0.05)	0.99	(0.06)	0.99	(0.07)

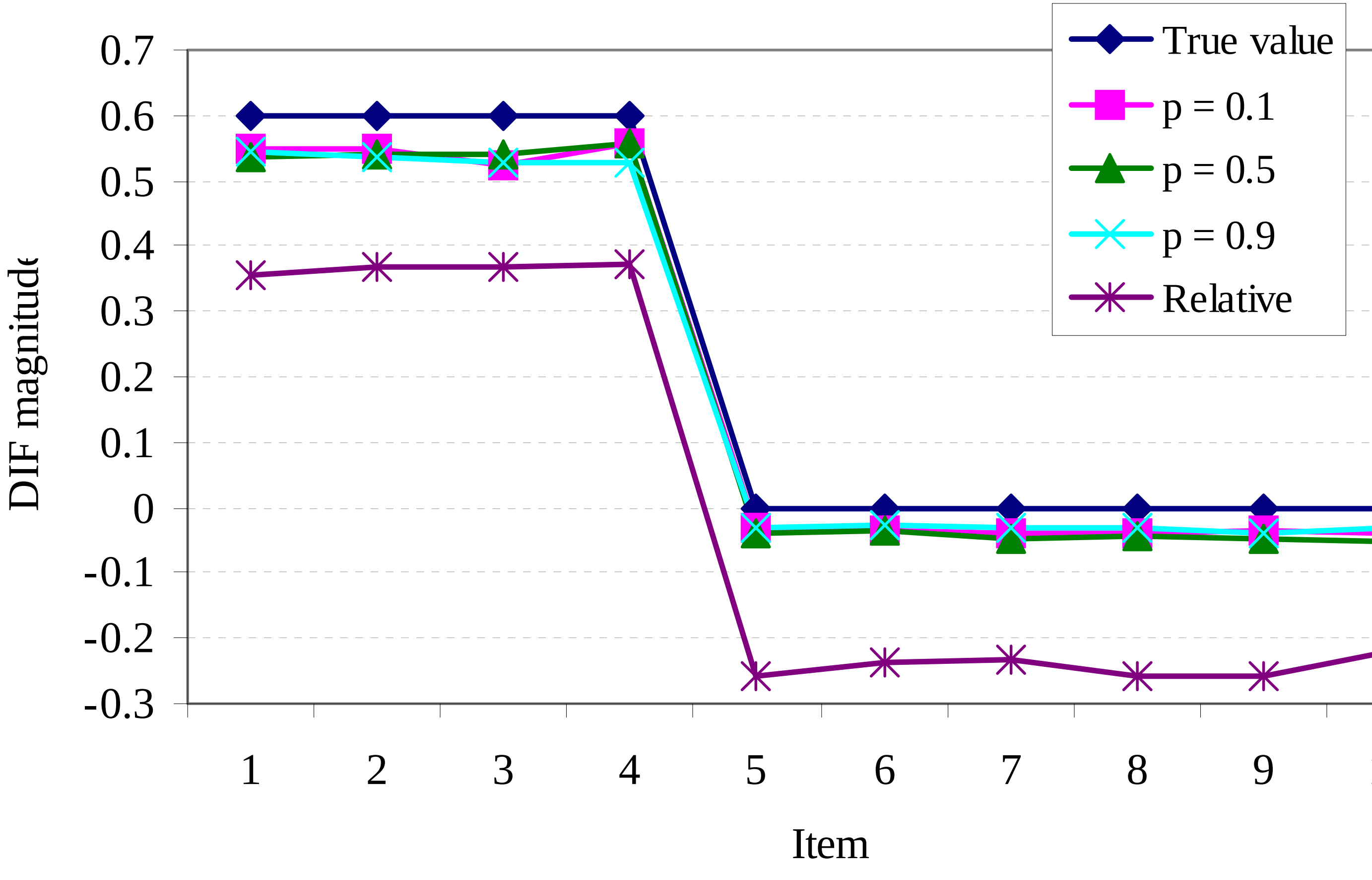
Simulation- Results 2 Cont.

Parameter Estimates: *n*=2000, Non-zero Mean DIF(0.6), 4 items

Parameter	True value	Mixutre DIF prior p=0.1 Mean	Mixutre DIF prior p=0.1 SD	Mixutre DIF prior p=0.5 Mean	Mixutre DIF prior p=0.5 SD	Mixutre DIF prior p=0.9 Mean	Mixutre DIF prior p=0.9 SD	Relative DIF Mean	Relative DIF SD
γ_1	**0.6**	**0.55**	**(0.14)**	**0.54**	**(0.15)**	**0.54**	**(0.15)**	**0.35**	**(0.11)**
γ_2	**0.6**	**0.55**	**(0.16)**	**0.54**	**(0.16)**	**0.54**	**(0.14)**	**0.37**	**(0.10)**
γ_3	**0.6**	**0.52**	**(0.15)**	**0.54**	**(0.16)**	**0.53**	**(0.13)**	**0.37**	**(0.10)**
γ_4	**0.6**	**0.56**	**(0.15)**	**0.56**	**(0.17)**	**0.53**	**(0.13)**	**0.37**	**(0.10)**
γ_5	0	-0.03	(0.11)	-0.04	(0.13)	-0.03	(0.10)	-0.26	(0.09)
γ_6	0	-0.03	(0.09)	-0.04	(0.11)	-0.03	(0.08)	-0.24	(0.09)
γ_7	0	-0.04	(0.10)	-0.05	(0.12)	-0.03	(0.09)	-0.23	(0.09)
γ_8	0	-0.04	(0.11)	-0.04	(0.13)	-0.03	(0.11)	-0.26	(0.10)
γ_9	0	-0.03	(0.11)	-0.05	(0.14)	-0.04	(0.09)	-0.26	(0.09)
γ_{10}	0	-0.04	(0.11)	-0.05	(0.13)	-0.03	(0.08)	-0.22	(0.09)
p		0.52	(0.05)	0.53	(0.05)	0.53	(0.05)		
$\sigma 2_1$	1	1.00	(0.04)	1.00	(0.04)	1.00	(0.04)	1.00	(0.04)
$\sigma 2_2$	1	1.01	(0.04)	1.01	(0.04)	1.01	(0.04)	1.00	(0.04)

Simulation- Results 2 Cont.

Parameter Estimates: n=2000, Non-zero Mean DIF(0.6), 4 items



Simulation- Results 3.

Parameter Estimates: n=2000, Zero Mean DIF(0.3,0.6), 4 items

Parameter	True value	Mixutre DIF						Relative DIF	
		prior p=0.1		prior p=0.5		prior p=0.9			
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
$\alpha 0$	0.3	0.30	(0.07)	0.29	(0.08)	0.30	(0.08)	0.31	(0.06)
β_1^{adj}	-1	-0.97	(0.09)	-0.96	(0.09)	-0.96	(0.08)	-1.00	(0.08)
β_2^{adj}	-1	-1.04	(0.08)	-1.03	(0.09)	-1.02	(0.08)	-1.00	(0.07)
β_3^{adj}	-1	-0.98	(0.07)	-1.00	(0.08)	-0.98	(0.07)	-1.00	(0.07)
β_4^{adj}	0	-0.01	(0.07)	-0.01	(0.07)	0.01	(0.07)	0.00	(0.07)
β_5^{adj}	0	0.00	(0.05)	0.01	(0.06)	-0.01	(0.06)	0.00	(0.07)
β_6^{adj}	0	0.00	(0.06)	-0.01	(0.05)	0.00	(0.06)	0.01	(0.06)
β_7^{adj}	0	0.00	(0.06)	-0.01	(0.05)	-0.01	(0.06)	-0.01	(0.07)
β_8^{adj}	1	1.00	(0.06)	1.00	(0.06)	0.99	(0.07)	1.00	(0.07)
β_9^{adj}	1	0.99	(0.06)	0.99	(0.06)	0.99	(0.06)	1.01	(0.07)
β_{10}^{adj}	1	1.01	(0.06)	1.01	(0.05)	0.99	(0.06)	1.00	(0.08)

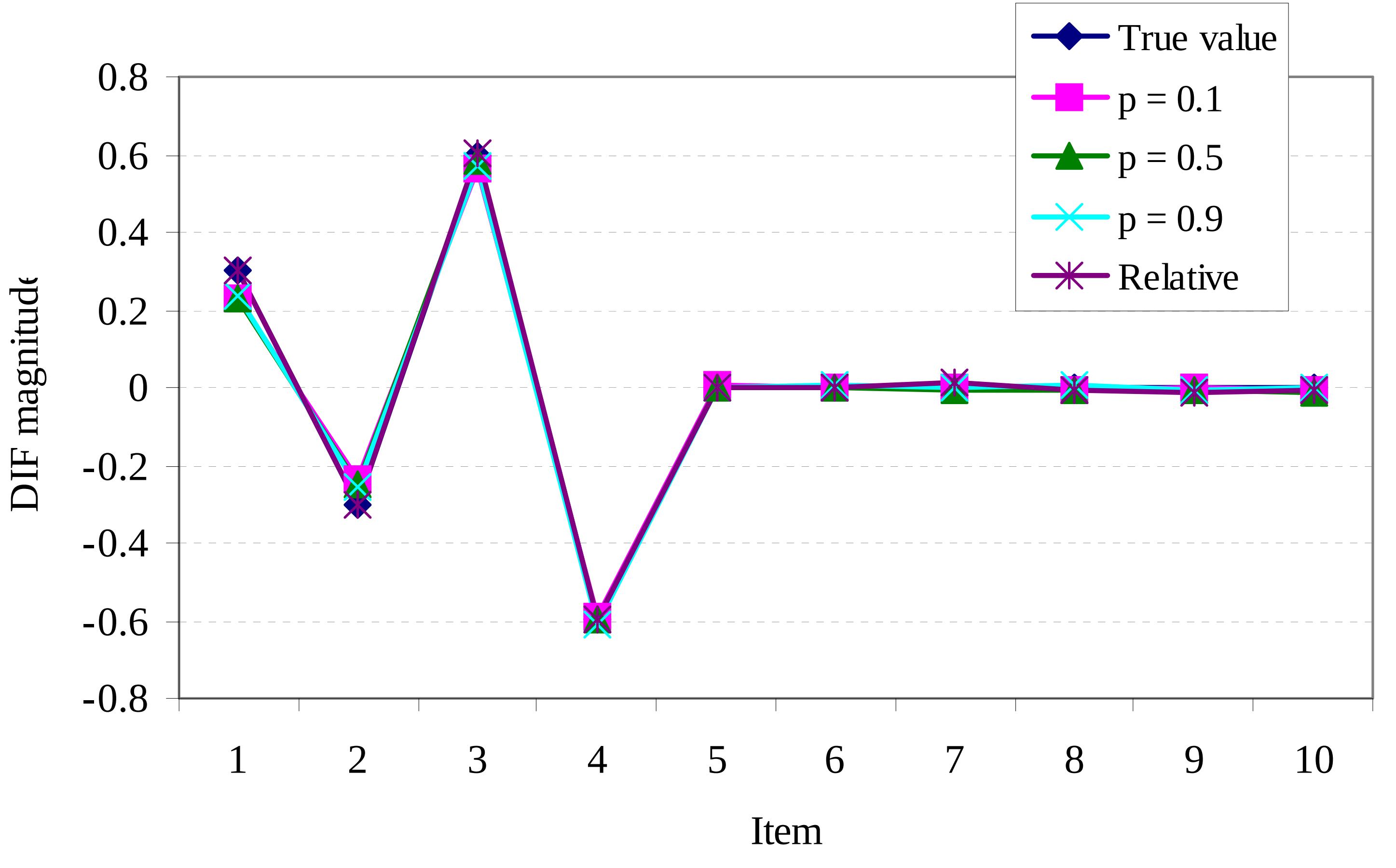
Simulation- Results 3 Cont.

Parameter Estimates: *n*=2000, Zero Mean DIF(0.3,0.6), 4 items

Parameter	True value	Mixutre DIF						Relative DIF	
		prior p=0.1		prior p=0.5		prior p=0.9			
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
γ_1	**0.3**	**0.23**	**(0.16)**	**0.23**	**(0.16)**	**0.24**	**(0.17)**	**0.30**	**(0.11)**
γ_2	**-0.3**	**-0.24**	**(0.16)**	**-0.25**	**(0.18)**	**-0.26**	**(0.18)**	**-0.30**	**(0.10)**
γ_3	**0.6**	**0.56**	**(0.12)**	**0.59**	**(0.10)**	**0.57**	**(0.12)**	**0.60**	**(0.10)**
γ_4	**-0.6**	**-0.59**	**(0.13)**	**-0.60**	**(0.15)**	**-0.61**	**(0.11)**	**-0.60**	**(0.10)**
γ_5	0	0.00	(0.04)	0.00	(0.04)	0.00	(0.04)	0.00	(0.10)
γ_6	0	0.00	(0.04)	0.00	(0.04)	0.00	(0.04)	0.00	(0.09)
γ_7	0	0.00	(0.04)	-0.01	(0.04)	0.00	(0.05)	0.01	(0.09)
γ_8	0	0.00	(0.04)	0.00	(0.06)	0.01	(0.06)	-0.01	(0.11)
γ_9	0	0.00	(0.05)	0.00	(0.04)	0.00	(0.03)	-0.01	(0.10)
γ_{10}	0	0.00	(0.03)	-0.01	(0.06)	0.00	(0.03)	-0.01	(0.12)
p		0.43	(0.06)	0.44	(0.06)	0.44	(0.06)		
$\sigma 2_1$	1	1.00	(0.04)	1.01	(0.04)	1.00	(0.04)	1.01	(0.04)
$\sigma 2_2$	1	1.00	(0.04)	1.01	(0.04)	1.01	(0.04)	1.00	(0.04)

Simulation- Results 3 Cont.

Parameter Estimates: n=2000, Zero Mean DIF(0.3,0.6), 4 items



Simulation- Results 4.

Parameter Estimates: n=2000, Zero Mean DIF(0.6), 4 items

Parameter	True value	Mixutre DIF						Relative DIF	
		prior p=0.1		prior p=0.5		prior p=0.9			
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
$\alpha 0$	0.3	0.30	(0.06)	0.30	(0.07)	0.30	(0.07)	0.31	(0.06)
β_1^{adj}	-1	-0.99	(0.07)	-0.99	(0.08)	-0.99	(0.07)	-1.01	(0.07)
β_2^{adj}	-1	-1.01	(0.07)	-1.00	(0.08)	-1.00	(0.07)	-1.01	(0.06)
β_3^{adj}	-1	-0.98	(0.07)	-1.00	(0.08)	-0.98	(0.07)	-1.02	(0.06)
β_4^{adj}	0	-0.01	(0.06)	-0.01	(0.07)	0.01	(0.07)	0.00	(0.06)
β_5^{adj}	0	0.00	(0.05)	0.01	(0.06)	-0.01	(0.06)	0.00	(0.06)
β_6^{adj}	0	0.00	(0.06)	-0.01	(0.05)	0.00	(0.06)	0.01	(0.07)
β_7^{adj}	0	0.00	(0.06)	-0.01	(0.06)	0.00	(0.06)	0.00	(0.07)
β_8^{adj}	1	1.00	(0.06)	1.00	(0.05)	0.99	(0.07)	1.01	(0.08)
β_9^{adj}	1	0.99	(0.06)	0.99	(0.06)	0.99	(0.06)	1.01	(0.07)
β_{10}^{adj}	1	1.01	(0.06)	1.01	(0.05)	0.99	(0.06)	1.00	(0.07)

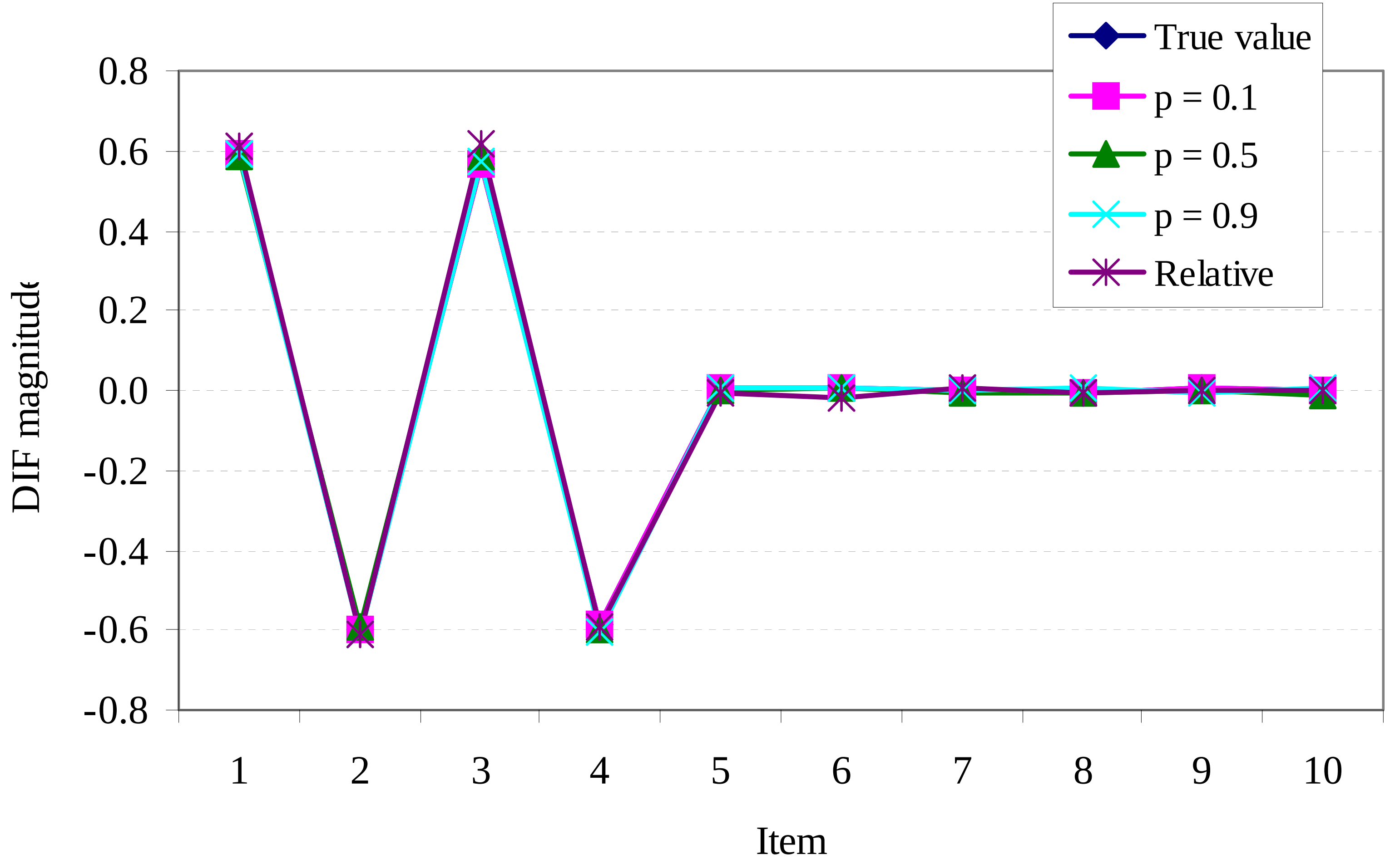
Simulation- Results 4 Cont.

Parameter Estimates: n=2000, Zero Mean DIF(0.6), 4 items

Parameter	True value	Mixutre DIF						Relative DIF	
		prior p=0.1		prior p=0.5		prior p=0.9			
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
γ_1	**0.6**	**0.59**	**(0.10)**	**0.59**	**(0.11)**	**0.59**	**(0.13)**	**0.61**	**(0.10)**
γ_2	**-0.6**	**-0.60**	**(0.13)**	**-0.59**	**(0.15)**	**-0.61**	**(0.15)**	**-0.61**	**(0.11)**
γ_3	**0.6**	**0.57**	**(0.12)**	**0.59**	**(0.10)**	**0.57**	**(0.11)**	**0.62**	**(0.09)**
γ_4	**-0.6**	**-0.59**	**(0.13)**	**-0.60**	**(0.13)**	**-0.61**	**(0.11)**	**-0.59**	**(0.09)**
γ_5	0	0.01	(0.04)	0.00	(0.05)	0.00	(0.05)	0.00	(0.10)
γ_6	0	0.00	(0.05)	0.00	(0.04)	0.01	(0.04)	-0.02	(0.10)
γ_7	0	0.00	(0.04)	-0.01	(0.04)	0.00	(0.06)	0.01	(0.10)
γ_8	0	0.00	(0.05)	0.00	(0.06)	0.01	(0.06)	0.00	(0.11)
γ_9	0	0.00	(0.05)	0.00	(0.05)	0.00	(0.04)	0.00	(0.10)
γ_{10}	0	0.00	(0.05)	-0.01	(0.07)	0.00	(0.04)	0.00	(0.11)
p		0.51	(0.04)	0.52	(0.04)	0.52	(0.04)		
$\sigma 2_1$	1	1.00	(0.04)	1.01	(0.04)	1.01	(0.04)	1.00	(0.04)
$\sigma 2_2$	1	1.00	(0.04)	1.01	(0.04)	1.01	(0.04)	1.01	(0.05)

Simulation- Results 4 Cont.

Parameter Estimates: n=2000, Zero Mean DIF(0.6), 4 items



Simulation- Results 5.

Parameter Estimates: n=2000, Non-zero Mean DIF(0.3, 0.6), 8 items

Parameter	True value	Mixutre DIF						Relative DIF	
		prior p=0.1		prior p=0.5		prior p=0.9			
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
$\square 0$	0.3	-0.09	(0.10)	-0.09	(0.10)	-0.08	(0.10)	-0.05	(0.06)
$\square_1^{adj}$	-1	-1.01	(0.06)	-1.01	(0.06)	-1.02	(0.07)	-1.00	(0.08)
$\square_2^{adj}$	-1	-1.01	(0.07)	-1.01	(0.07)	-1.01	(0.07)	-1.01	(0.07)
$\square_3^{adj}$	-1	-1.02	(0.07)	-1.02	(0.07)	-1.03	(0.07)	-1.00	(0.06)
$\square_4^{adj}$	0	-0.02	(0.07)	-0.02	(0.07)	-0.02	(0.07)	0.00	(0.07)
$\square_5^{adj}$	0	0.02	(0.08)	0.02	(0.08)	0.03	(0.07)	0.00	(0.07)
$\square_6^{adj}$	0	0.02	(0.07)	0.02	(0.07)	0.02	(0.07)	0.01	(0.07)
$\square_7^{adj}$	0	0.01	(0.06)	0.01	(0.06)	0.03	(0.07)	-0.01	(0.07)
$\square_8^{adj}$	1	1.03	(0.07)	1.03	(0.07)	1.03	(0.07)	1.00	(0.08)
$\square_9^{adj}$	1	1.00	(0.08)	1.00	(0.08)	0.97	(0.09)	1.00	(0.07)
$\square_{10}^{adj}$	1	0.99	(0.08)	0.99	(0.08)	0.98	(0.08)	1.00	(0.07)

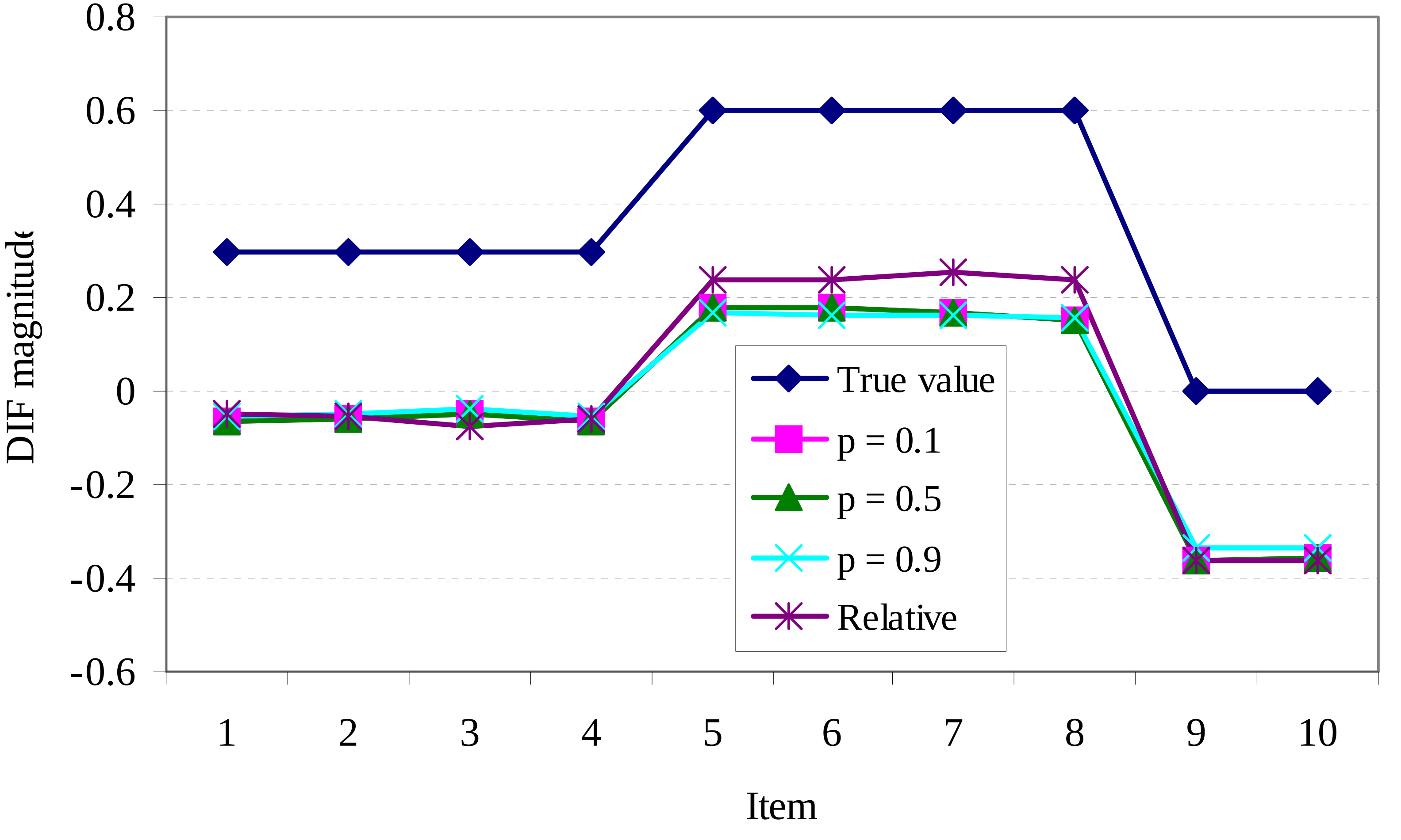
Simulation- Results 5 Cont.

Parameter Estimates: *n*=2000, Non-zero Mean DIF(0.3, 0.6), 8 items

Parameter	True value	Mixutre DIF						Relative DIF	
		prior p=0.1		prior p=0.5		prior p=0.9			
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
$\square_1$	0.3	-0.07	(0.10)	-0.07	(0.10)	-0.06	(0.09)	-0.05	(0.11)
$\square_2$	0.3	-0.06	(0.09)	-0.06	(0.09)	-0.05	(0.07)	-0.06	(0.10)
$\square_3$	0.3	-0.05	(0.10)	-0.05	(0.10)	-0.04	(0.08)	-0.08	(0.10)
$\square_4$	0.3	-0.06	(0.09)	-0.06	(0.09)	-0.05	(0.08)	-0.06	(0.09)
$\square_5$	0.6	0.18	(0.14)	0.18	(0.14)	0.17	(0.14)	0.24	(0.10)
$\square_6$	0.6	0.18	(0.14)	0.18	(0.14)	0.16	(0.14)	0.24	(0.09)
$\square_7$	0.6	0.17	(0.15)	0.17	(0.15)	0.16	(0.16)	0.25	(0.10)
$\square_8$	0.6	0.15	(0.14)	0.15	(0.14)	0.16	(0.16)	0.24	(0.10)
$\square_9$	0	-0.36	(0.19)	-0.36	(0.19)	-0.33	(0.17)	-0.36	(0.11)
$\square_{10}$	0	-0.35	(0.18)	-0.35	(0.18)	-0.33	(0.18)	-0.36	(0.11)
p	-	0.50	(0.10)	0.50	(0.10)	0.47	(0.10)		
$\square 2_1$	1	1.01	(0.04)	1.01	(0.04)	1.01	(0.04)	1.00	(0.04)
$\square 2_2$	1	1.00	(0.04)	1.00	(0.04)	0.99	(0.04)	1.00	(0.04)

Simulation- Results 5 Cont.

Parameter Estimates: *n*=2000, Non-zero Mean DIF(0.3, 0.6), 8 items



Simulation- Results 6.

Parameter Estimates: *n*=2000, Non-zero Mean DIF(0.6), 8 items

Parameter	True value	Mixutre DIF						Relative DIF	
		prior p=0.1		prior p=0.5		prior p=0.9			
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
$/_0$	0.3	-0.29	(0.06)	-0.29	(0.06)	-0.30	(0.06)	-0.17	(0.05)
$/_1^{adj}$	-1	-1.00	(0.06)	-1.00	(0.06)	-1.00	(0.05)	-1.00	(0.08)
$/_2^{adj}$	-1	-1.00	(0.06)	-1.00	(0.06)	-0.99	(0.06)	-1.00	(0.07)
$/_3^{adj}$	-1	-1.00	(0.05)	-1.00	(0.05)	-1.00	(0.05)	-1.00	(0.07)
$/_4^{adj}$	0	0.00	(0.05)	0.00	(0.05)	0.01	(0.05)	0.00	(0.07)
$/_5^{adj}$	0	0.00	(0.05)	0.00	(0.05)	0.00	(0.05)	0.00	(0.07)
$/_6^{adj}$	0	-0.01	(0.04)	-0.01	(0.04)	0.01	(0.05)	0.01	(0.06)
$/_7^{adj}$	0	0.00	(0.05)	0.00	(0.05)	0.01	(0.05)	-0.01	(0.07)
$/_8^{adj}$	1	1.01	(0.06)	1.01	(0.06)	1.00	(0.05)	1.00	(0.07)
$/_9^{adj}$	1	1.00	(0.07)	1.00	(0.07)	0.98	(0.07)	1.01	(0.07)
$/_{10}^{adj}$	1	1.00	(0.08)	1.00	(0.08)	0.99	(0.08)	1.00	(0.08)

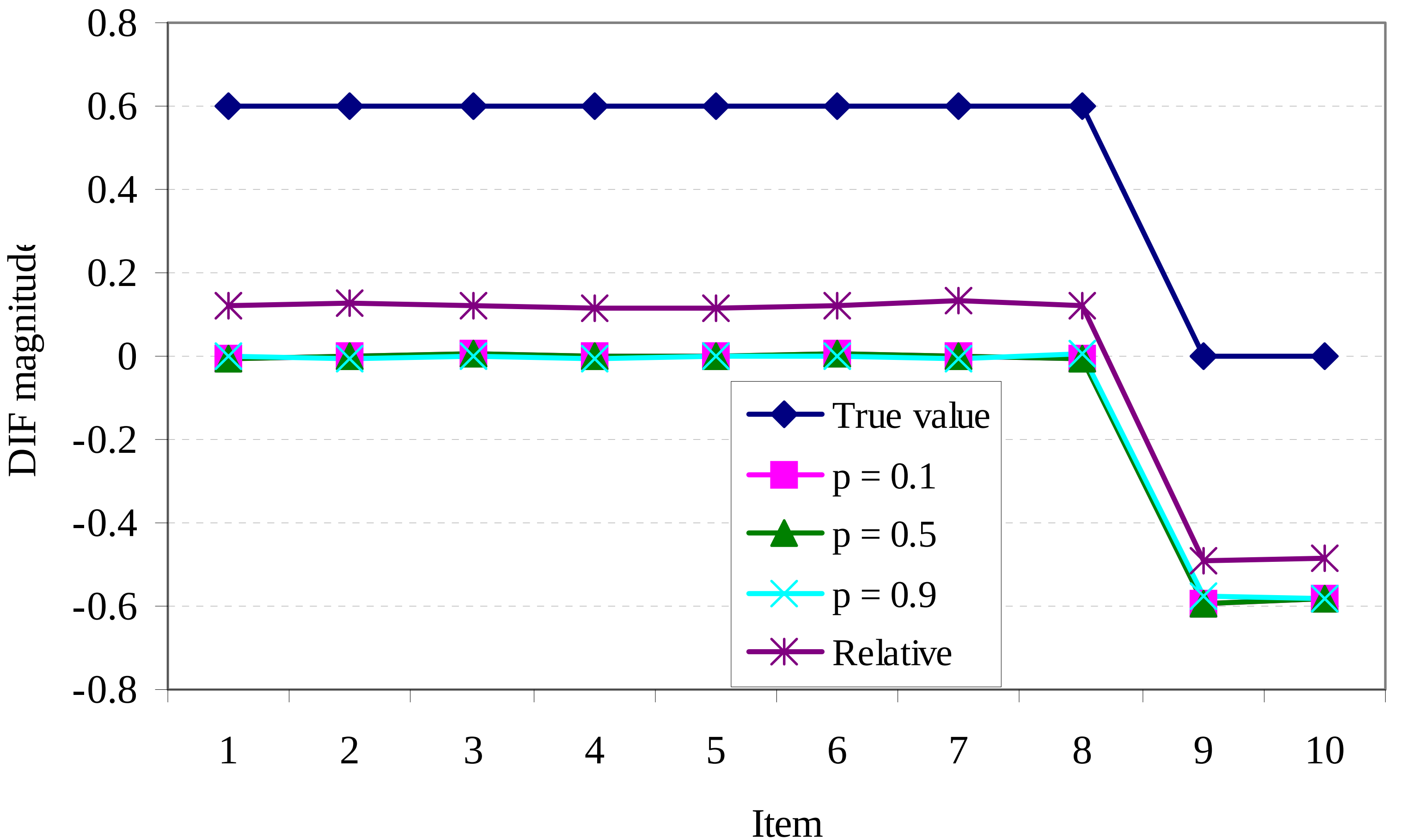
Simulation- Results 6 Cont.

Parameter Estimates: *n*=2000, Non-zero Mean DIF(0.6), 8 items

Parameter	True value	Mixutre DIF						Relative DIF	
		prior p=0.1		prior p=0.5		prior p=0.9			
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
$/_1$	0.6	-0.01	(0.05)	-0.01	(0.05)	0.00	(0.05)	0.12	(0.10)
$/_2$	0.6	0.00	(0.03)	0.00	(0.03)	0.00	(0.03)	0.13	(0.09)
$/_3$	0.6	0.00	(0.03)	0.00	(0.03)	0.00	(0.03)	0.12	(0.10)
$/_4$	0.6	0.00	(0.03)	0.00	(0.03)	-0.01	(0.03)	0.12	(0.09)
$/_5$	0.6	0.00	(0.04)	0.00	(0.04)	0.00	(0.02)	0.11	(0.10)
$/_6$	0.6	0.01	(0.04)	0.01	(0.04)	0.00	(0.03)	0.12	(0.09)
$/_7$	0.6	0.00	(0.02)	0.00	(0.02)	0.00	(0.04)	0.13	(0.09)
$/_8$	0.6	0.00	(0.03)	0.00	(0.03)	0.00	(0.04)	0.12	(0.11)
$/_9$	0	-0.59	(0.13)	-0.59	(0.13)	-0.57	(0.11)	-0.49	(0.10)
$/_{10}$	0	-0.58	(0.13)	-0.58	(0.13)	-0.58	(0.13)	-0.49	(0.12)
p		0.32	(0.03)	0.32	(0.03)	0.32	(0.04)		
$/2_1$	1	1.00	(0.04)	1.00	(0.04)	1.00	(0.05)	1.01	(0.04)
$/2_2$	1	1.00	(0.04)	1.00	(0.04)	1.00	(0.04)	1.00	(0.04)

Simulation- Results 6 Cont.

Parameter Estimates: *n*=2000, Non-zero Mean DIF(0.6), 8 items



Simulation- Results 7.

Parameter Estimates: n=2000, Zero Mean DIF(0.3, 0.6), 8 items

Parameter	True	Mixutre DIF						Relative DIF	
		prior p=0.1		prior p=0.5		prior p=0.9			
	value	Mean	SD	Mean	SD	Mean	SD	Mean	SD
$\Xi 0$	0.3	0.32	(0.18)	0.29	(0.15)	0.29	(0.16)	0.31	(0.06)
Ξ_1^{adj}	-1	-0.99	(0.07)	-1.00	(0.08)	-0.99	(0.08)	-1.01	(0.07)
Ξ_2^{adj}	-1	-0.99	(0.07)	-0.98	(0.08)	-0.98	(0.07)	-1.00	(0.06)
Ξ_3^{adj}	-1	-1.01	(0.07)	-1.02	(0.08)	-1.01	(0.07)	-1.01	(0.07)
Ξ_4^{adj}	0	-0.01	(0.07)	0.00	(0.07)	0.00	(0.07)	0.01	(0.07)
Ξ_5^{adj}	0	0.01	(0.06)	0.01	(0.06)	0.01	(0.07)	0.00	(0.07)
Ξ_6^{adj}	0	0.01	(0.06)	0.02	(0.06)	0.00	(0.06)	0.00	(0.06)
Ξ_7^{adj}	0	-0.01	(0.07)	-0.01	(0.07)	-0.01	(0.07)	0.00	(0.07)
Ξ_8^{adj}	1	0.99	(0.07)	1.00	(0.07)	0.99	(0.07)	1.00	(0.07)
Ξ_9^{adj}	1	1.00	(0.08)	1.00	(0.07)	0.99	(0.08)	1.00	(0.08)
Ξ_{10}^{adj}	1	1.00	(0.07)	0.99	(0.08)	1.00	(0.08)	1.00	(0.07)

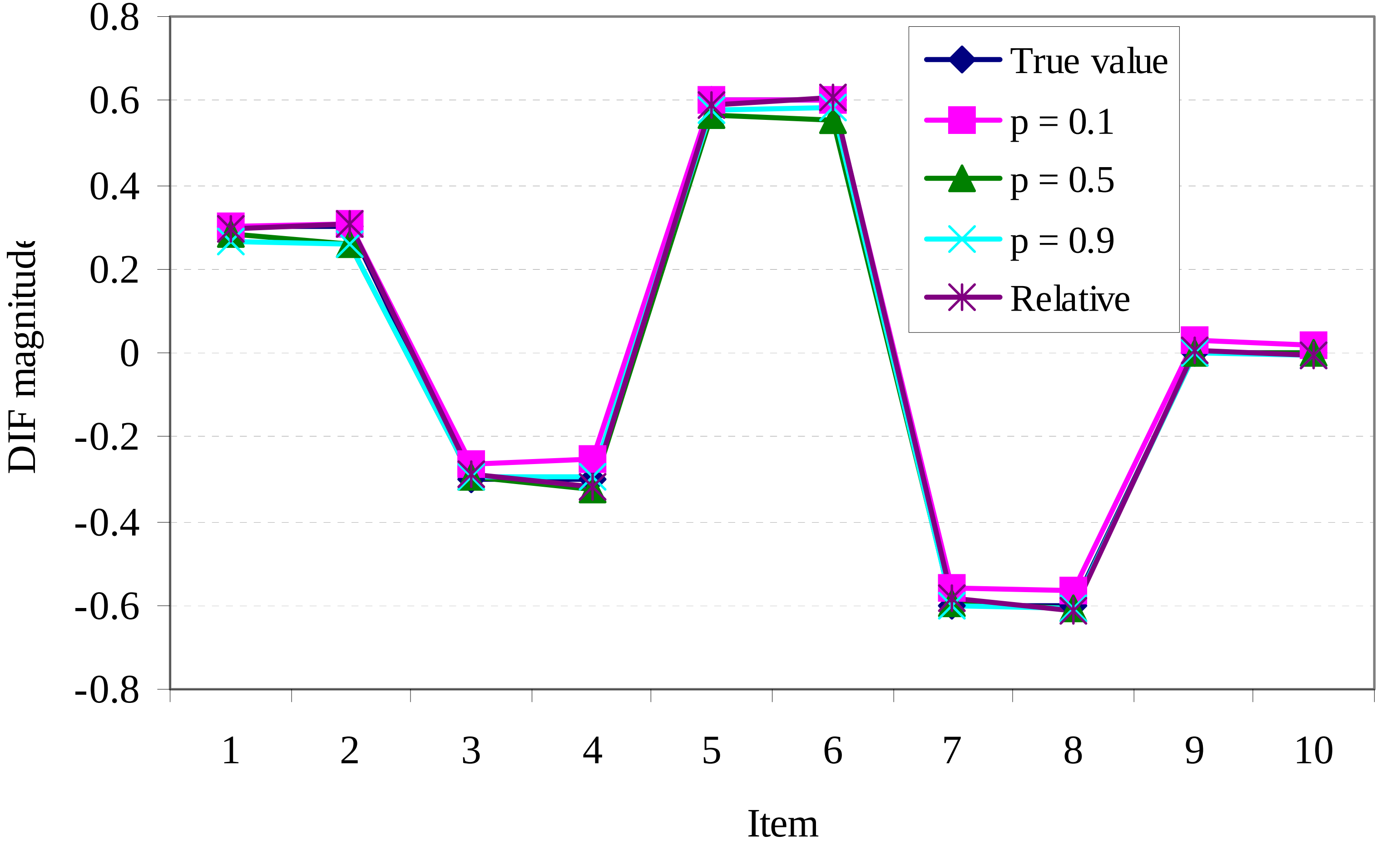
Simulation- Results 7 Cont.

Parameter Estimates: n=2000, Zero Mean DIF(0.3, 0.6), 8 items

Parameter	True value	Mixutre DIF						Relative DIF	
		prior p=0.1		prior p=0.5		prior p=0.9			
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
Ξ_1	0.3	0.30	(0.17)	0.28	(0.17)	0.27	(0.17)	0.30	(0.11)
Ξ_2	0.3	0.31	(0.19)	0.26	(0.15)	0.26	(0.17)	0.31	(0.10)
Ξ_3	-0.3	-0.27	(0.20)	-0.29	(0.19)	-0.30	(0.19)	-0.29	(0.10)
Ξ_4	-0.3	-0.25	(0.19)	-0.33	(0.19)	-0.30	(0.18)	-0.32	(0.11)
Ξ_5	0.6	0.60	(0.21)	0.57	(0.18)	0.58	(0.19)	0.59	(0.10)
Ξ_6	0.6	0.60	(0.19)	0.56	(0.19)	0.58	(0.19)	0.61	(0.10)
Ξ_7	-0.6	-0.56	(0.21)	-0.60	(0.19)	-0.60	(0.19)	-0.59	(0.09)
Ξ_8	-0.6	-0.57	(0.22)	-0.61	(0.19)	-0.61	(0.19)	-0.61	(0.09)
Ξ_9	0	0.03	(0.14)	0.00	(0.14)	0.00	(0.14)	0.00	(0.11)
Ξ_{10}	0	0.02	(0.16)	0.00	(0.15)	-0.01	(0.14)	-0.01	(0.09)
p		0.74	(0.04)	0.75	(0.03)	0.74	(0.04)		
$\Xi2_1$	1	1.00	(0.03)	1.00	(0.04)	1.01	(0.04)	1.00	(0.04)
$\Xi2_2$	1	1.00	(0.04)	1.00	(0.04)	1.00	(0.04)	1.00	(0.05)

Simulation- Results 7 Cont.

Parameter Estimates: *n*=2000, Zero Mean DIF(0.3, 0.6), 8 items



Simulation- Results 8.

Parameter Estimates: n=2000, Zero Mean DIF(0.6), 8 items

Parameter	True value	Mixutre DIF prior p=0.1 Mean	Mixutre DIF prior p=0.1 SD	Mixutre DIF prior p=0.5 Mean	Mixutre DIF prior p=0.5 SD	Mixutre DIF prior p=0.9 Mean	Mixutre DIF prior p=0.9 SD	Relative DIF Mean	Relative DIF SD
$(_0$	0.3	0.35	(0.32)	0.35	(0.32)	0.33	(0.31)	0.29	(0.06)
$(_1^{adj}$	-1	-1.00	(0.07)	-1.00	(0.07)	-1.00	(0.07)	-1.00	(0.07)
$(_2^{adj}$	-1	-1.01	(0.07)	-1.01	(0.07)	-1.00	(0.07)	-1.00	(0.07)
$(_3^{adj}$	-1	-1.00	(0.07)	-1.00	(0.07)	-1.00	(0.07)	-1.01	(0.07)
$(_4^{adj}$	0	-0.01	(0.06)	-0.01	(0.06)	0.00	(0.07)	0.00	(0.06)
$(_5^{adj}$	0	0.00	(0.06)	0.00	(0.06)	0.00	(0.06)	0.01	(0.07)
$(_6^{adj}$	0	-0.02	(0.06)	-0.02	(0.06)	0.00	(0.06)	0.00	(0.06)
$(_7^{adj}$	0	0.01	(0.06)	0.01	(0.06)	0.01	(0.07)	0.01	(0.07)
$(_8^{adj}$	1	1.01	(0.07)	1.01	(0.07)	0.99	(0.06)	1.00	(0.07)
$(_9^{adj}$	1	1.01	(0.07)	1.01	(0.07)	0.99	(0.07)	0.99	(0.07)
$(_{10}^{adj}$	1	1.00	(0.08)	1.00	(0.08)	1.00	(0.08)	1.00	(0.06)

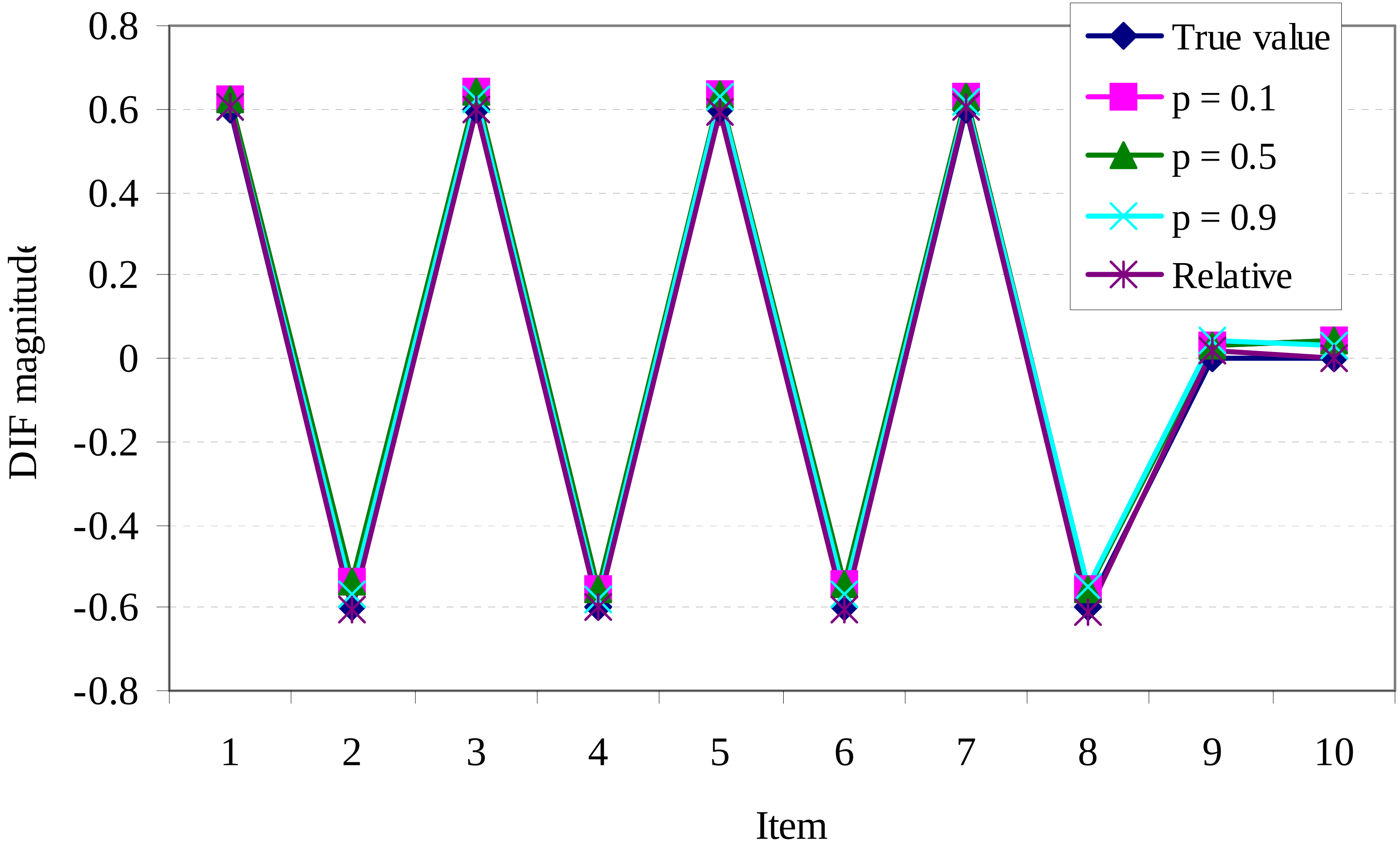
Simulation- Results 8 Cont.

Parameter Estimates: *n*=2000, Zero Mean DIF(0.6), 8 items

Parameter	True value	Mixutre DIF prior p=0.1 Mean	SD	prior p=0.5 Mean	SD	prior p=0.9 Mean	SD	Relative DIF Mean	SD
($_{1}$	0.6	0.62	(0.34)	0.62	(0.34)	0.61	(0.33)	0.60	(0.09)
($_{2}$	-0.6	-0.53	(0.31)	-0.53	(0.31)	-0.57	(0.34)	-0.60	(0.12)
($_{3}$	0.6	0.64	(0.33)	0.64	(0.33)	0.63	(0.32)	0.60	(0.09)
($_{4}$	-0.6	-0.55	(0.33)	-0.55	(0.33)	-0.58	(0.33)	-0.60	(0.10)
($_{5}$	0.6	0.64	(0.35)	0.64	(0.35)	0.63	(0.33)	0.59	(0.09)
($_{6}$	-0.6	-0.54	(0.33)	-0.54	(0.33)	-0.57	(0.34)	-0.61	(0.09)
($_{7}$	0.6	0.63	(0.33)	0.63	(0.33)	0.62	(0.35)	0.60	(0.09)
($_{8}$	-0.6	-0.55	(0.31)	-0.55	(0.31)	-0.55	(0.33)	-0.61	(0.10)
($_{9}$	0	0.03	(0.32)	0.03	(0.32)	0.04	(0.31)	0.02	(0.10)
($_{10}$	0	0.04	(0.32)	0.04	(0.32)	0.03	(0.30)	0.00	(0.09)
p		0.76	(0.04)	0.76	(0.04)	0.76	(0.04)		
(2_1	1	1.00	(0.04)	1.00	(0.04)	1.00	(0.05)	1.00	(0.04)
(2_2	1	1.00	(0.04)	1.00	(0.04)	1.00	(0.04)	1.01	(0.04)

Simulation- Results 8 Cont.

Parameter Estimates: *n*=2000, Zero Mean DIF(0.6), 8 items



Simulation- Results 9.

Parameter Estimates: n=2000, Zero Mean DIF(0.6), 10 items

Parameter	True value	Mixutre DIF				Relative DIF	
		prior p=0.1		prior p=0.9			
		Mean	SD	Mean	SD	Mean	SD
$\Xi 0$	0.3	0.43	(0.47)	0.42	(0.47)	0.30	(0.06)
Ξ_1^{adj}	-1	-0.99	(0.07)	-0.99	(0.07)	-1.01	(0.07)
Ξ_2^{adj}	-1	-0.99	(0.06)	-0.99	(0.06)	-1.01	(0.07)
Ξ_3^{adj}	-1	-1.00	(0.07)	-1.00	(0.07)	-1.00	(0.07)
Ξ_4^{adj}	0	-0.01	(0.06)	-0.01	(0.06)	-0.01	(0.07)
Ξ_5^{adj}	0	0.01	(0.06)	0.01	(0.07)	-0.01	(0.07)
Ξ_6^{adj}	0	-0.01	(0.06)	-0.01	(0.06)	0.01	(0.06)
Ξ_7^{adj}	0	0.01	(0.06)	0.01	(0.06)	0.00	(0.06)
Ξ_8^{adj}	1	1.00	(0.07)	1.00	(0.07)	1.01	(0.06)
Ξ_9^{adj}	1	0.99	(0.07)	0.99	(0.07)	1.00	(0.07)
Ξ_{10}^{adj}	1	0.99	(0.06)	0.99	(0.06)	1.00	(0.08)

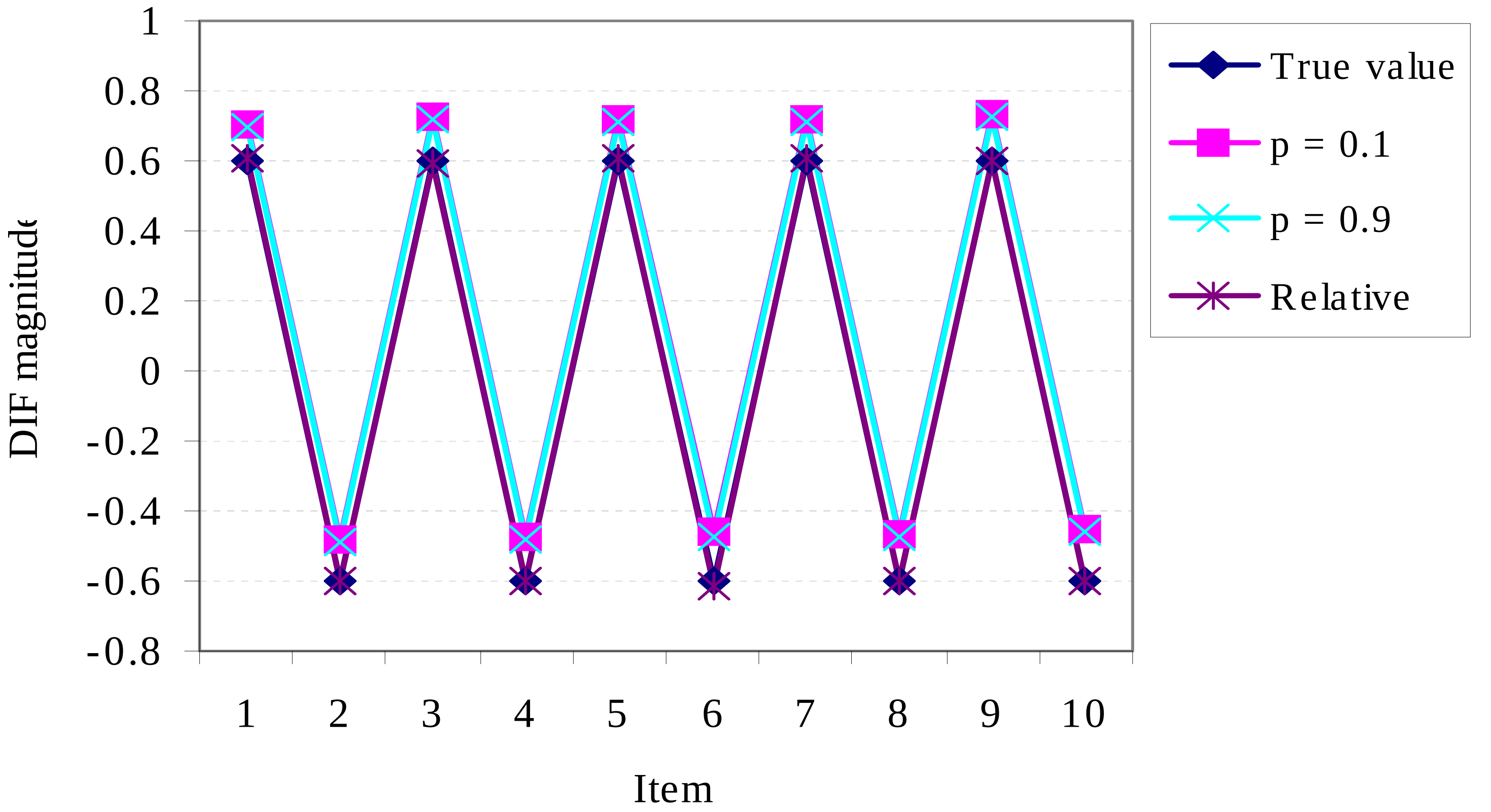
Simulation- Results 9 Cont.

Parameter Estimates: *n*=2000, Zero Mean DIF(0.6), 10 items

Parameter	True value	Mixutre DIF				Relative DIF	
		prior p=0.1		prior p=0.9			
		Mean	SD	Mean	SD	Mean	SD
$\Xi 0$	0.3	0.43	(0.47)	0.42	(0.47)	0.30	(0.06)
Ξ_1	0.6	0.71	(0.46)	0.70	(0.46)	0.61	(0.10)
Ξ_2	-0.6	-0.48	(0.48)	-0.49	(0.48)	-0.60	(0.11)
Ξ_3	0.6	0.73	(0.47)	0.72	(0.47)	0.59	(0.10)
Ξ_4	-0.6	-0.47	(0.48)	-0.48	(0.48)	-0.60	(0.11)
Ξ_5	0.6	0.72	(0.49)	0.71	(0.49)	0.61	(0.10)
Ξ_6	-0.6	-0.46	(0.47)	-0.47	(0.47)	-0.62	(0.10)
Ξ_7	0.6	0.72	(0.48)	0.71	(0.48)	0.61	(0.10)
Ξ_8	-0.6	-0.47	(0.47)	-0.48	(0.47)	-0.60	(0.10)
Ξ_9	0.6	0.73	(0.50)	0.72	(0.50)	0.60	(0.11)
Ξ_{10}	-0.6	-0.45	(0.47)	-0.46	(0.47)	-0.60	(0.11)
p	-	0.65	(0.05)	0.65	(0.05)		
$\Xi 2_1$	1	1.01	(0.04)	1.01	(0.04)	1.00	(0.04)
$\Xi 2_2$	1	1.00	(0.03)	1.00	(0.03)	1.01	(0.03)

Simulation- Results 9 Cont.

Parameter Estimates: *n*=2000, Zero Mean DIF(0.6), 10 items



Findings- results

- Small sample size conditions produce worse results
- Zero-mean DIF conditions for 4 items
  - Mixture and relative DIF models are recovered very well
- Nonzero-mean DIF conditions
  - Mixture model produced better results
- Mixture DIF model can be useful in some situations.
- Relative DIF model cannot provide correct scaling of DIF magnitude estimates.

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Thank you